Technical Notes

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Three-Dimensional Wing Boundary Layer Calculated with Eight Different Methods

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Introduction

LITTLE was published in boundary-layer research following the 1968 Stanford Conference[1] until East's "Trondheim Trials" report[2] appeared. The Trondheim Trials session of the Euromech 60 Colloquium[3] enabled the then state-of-the-art in three-dimensional turbulent boundary-layer computation to be assessed by comparing the performance of different methods on common test cases. This was similar to the comparison between calculation and experiment for two-dimensional flow which took place at the Stanford Conference, but its scope was more limited. The report was well received and may have initiated some of the new efforts in three-dimensional turbulent boundary-layer research now going on.[4-7]

As a response to the new interest, The Eurovisc Committee of National Representatives set up a permanent working party to encourage and carry out evaluation of calculation methods for shear flows. The first meeting of the Working Party was a workshop held at FFA (The Aeronautical Research Institute of Sweden), and the main objective of the meeting was to discuss results obtained in applying eight different boundary-layer calculation methods to the same three-dimensional wing test case. This Note describes the test case and gives a selection of the most important calculated results.

Configuration

The wing planform is trapezoidal, the camber surface is planar and the section is NACA 64A010 perpendicular to the quarter chord line, which is swept 35 deg. The taper ratio is 0.4, and the aspect ratio is 4.

Inviscid Flow

The wing pressures have been determined both at FFA and at the NAE in Canada.[8] However, for use as a boundary-layer test case, inviscid velocity components are needed. To provide the velocities and also to ensure sufficiently dense and even coverage of wetted surfaces, the inviscid conditions were calculated, using a higher-order panel method.[9] Flow Mach number in the calculations corresponded to the lowest tested, i.e., 0.5. Calculated inviscid pressure coefficients were found to differ in level somewhat from the measurements,[10] but the pressure gradients are considered to have been reproduced well enough for reliable boundary-layer determination. At zero incidence the calculated isobars follow the generators closely, but at 8 deg they do so only over the rear half of the wing.[11] Pressure gradients are mild and smoothly distributed at both incidences.

Sensitivity of the inviscid flow to the number and distribution of panels was also investigated in detail.[10] The arrangement adopted for the test case yielded a grid-invariant solution aft of the 10% generator. However, the nose region was excluded from the boundary-layer calculations proper. Instead, the conditions here were estimated using simplifying assumptions and the results used to provide starting values.

Initial Conditions

Flow Reynolds number was 7×10^6 based on mean chord, which is in the middle of the experimental range.[8] To minimize the amount of information needed to define conditions on the initial line (15% chord), assumed two-dimensional, only one starting velocity profile was prepared. This was to be used at all span stations and at both incidences treated—zero and 8 deg. The boundary layer was to be computed for the upper surface only.

After estimating a mean effective streamwise development Reynolds number up to the starting line, the skin friction coefficient was computed using a correlation of flat plate experimental data.[12] The profile itself was then derived using an analytical approximation to the Thompson family[13] and compressibility was allowed for with the van Driest transformation in the form recommended by Bradshaw.[14]

Boundary-Layer Methods

Developers of calculation methods were now contacted and invited to compute the test case. Eight groups responded favorably, were sent the case definition on cards, and subsequently submitted results to FFA where they were analyzed and compared. Table 1 shows which workers participated, the three-character identifiers of the results, the corresponding symbols used in the figures, and the method descriptors—I or D for "integral" or "differential," respectively.

Two restrictions should be noted. The B+B and R+R calculations assumed incompressible flow and the BRA results were calculated assuming quasi-two-dimensional flow as on an ideal trapezoidal wing. The turbulence was simulated as follows: eddy viscosity (C+A, KOR, MCL and R+R), entrainment (C+A, SMI, and STO), one equation (B+B and BRA), and two equation (R+R) models. More details may be found in the full report.[11]

Results

Only a small part of the results obtained is reported herein. Wall crossflow angle β_w on the mid-semispan section is shown in Fig. 1. This is the angle between the external inviscid streamline direction and the wall limiting streamline direction. Appreciable three-dimensionality might be defined as $|\beta_w|>1$ deg, say, related to the accuracy with which β_w can be measured, in which case the boundary layer is three-dimensional essentially from the starting line at 8 deg incidence, but only downstream of 60% chord at zero incidence. On the other hand, effective three-dimensionality probably sets in at β_w values at least an order of magnitude greater than this, where appreciable coupling between the streamwise and crossflow momentum equations first starts; on this measure, the boundary layer is hardly three-dimensional. Together with the well-behaved external flow already noted, this suggests that the test case should not be regarded as severe. Nevertheless, the scatter band width produced by superimposing the different results for β_w is as much as 20%.

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Table 1 Results' sources and key to figures

Participants	Label	Symbol	Method
S. H. Boelsma and B. van den Berg	B+B	●	D
P. Bradshaw	BRA	■	D
J. Cousteix and B. Aupoix	C+A	△	I
W. Kordulla	KOR	▲	D
J. D. McLean	MCL	▽	D
A. K. Rastogi and W. Rodi	R+R	▼	D
P. D. Smith	SMI	○	I
H.-W. Stock	STO	□	I

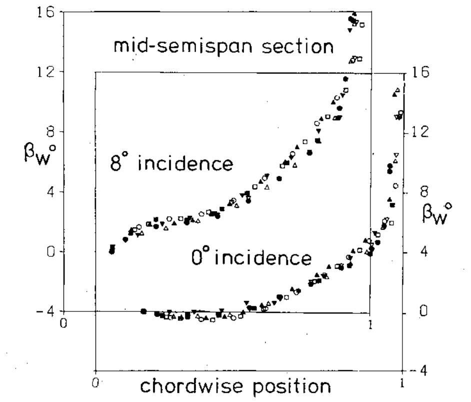

Fig. 1 Chordwise crossflow angle variation.

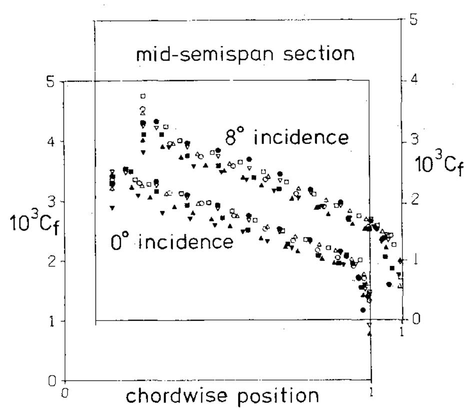

Fig. 2 Chordwise total skin friction coefficient variation.

Figure 2 shows variation of skin friction coefficient C_f on the mid-semispan section, which also produced 20% scatter. The shape factor H based on streamwise integral thicknesses is given in Fig. 3 for the same section. The incompressible R + R results can be raised to within 2 or 3% of the average of the rest[11] by applying the Spence formula[15] for the variation of H with Mach numbers. Nevertheless, the scatter in H is still very large, considering the limits to normal H variation.

Conclusions

The external flow was well behaved and the boundary layer was for the most part not very three-dimensional. Its calculation ought not to present very great difficulties. Despite this, eight current three-dimensional turbulent boundary-layer calculation methods gave somewhat disappointing agreement. Limiting streamline angle β_w and skin friction coefficient C_f both showed 20% scatter, although the integral methods and the quasi-two-dimensional method appeared to fare no worse than the more sophisticated approaches. It must be concluded that much remains to be done in three-dimensional turbulent boundary-layer computation to reach a consensus for even the most straightforward cases.

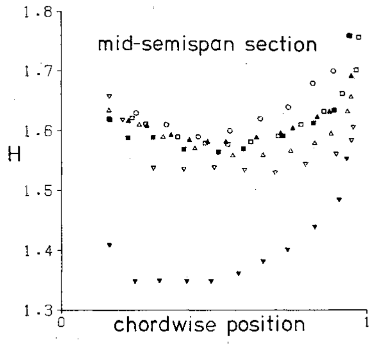

Fig. 3 Chordwise shape parameter variation at 8 deg incidence.

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Transonic Solution for Nieuwland Profiles Using Spline Interpolation

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I. Introduction

USING the indirect hodograph method, Nieuwland[1] obtained exact shock-free solutions for a number of quasielliptical airfoil sections. But the hodograph method has the disadvantage that the body shape cannot be prescribed a priori, so the boundary condition has to be satisfied at unknown boundaries.

The direct problem where the profile shape and the freestream Mach number M_∞ (<1) are prescribed is of much practical interest. We consider here the case of a thin symmetric profile at zero incidence. The integral equation formulation of this problem[2] leads to a two-dimensional nonlinear singular integral equation for the unknown u component of the velocity parallel to the freestream direction. This is known as the integral equation of Oswatitsch. Niyogi[3,4] obtained an approximate solution of the above integral equation for shock-free profile flow where the transonic solution is expressed in terms of the corresponding linearized Prandtl solution. Comparison with other results indicated good agreement.

However, it turns out that the profile geometry of most body shapes of practical interest is given numerically. In computing the linearized Prandtl solution, the profile slope is needed, which then has to be evaluated numerically. Furthermore, a singular integral remains to be computed numerically. In general, this leads to loss of accuracy. To overcome this, in the present work, the profile shape has been represented by cubic splines, which has the advantage that the profile slopes are derived with adequate accuracy. Moreover, the integration needed for evaluating the linearized Prandtl solution can be performed analytically.

In the present work, results have been computed for a number of symmetrical quasielliptical Nieuwland profiles at zero incidence, for which exact solutions are known.[1] An edge correction has been used in the linearized solution. Excellent agreement has been found in all cases.

II. Formulation of the Problem

We consider steady inviscid transonic flow past a thin symmetric profile at zero incidence, with subsonic freestream Mach number $M_\infty < 1$. According to integral equation formulation, the flow problem in the shock-free case is governed by the following two-dimensional nonlinear singular integral equation[2,3]:

$$U(X,Y) = U_p(X,Y) + \frac{U^2(X,Y)}{4} - \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times \frac{U^2(\xi,\eta)}{2}\,\frac{(\xi-x)^2-(\eta-y)^2}{[(\xi-x)^2+(\eta-y)^2]^2}\,\mathrm{d}\xi\mathrm{d}\eta \tag{1}$$

Here, reduced rectangular Cartesian coordinates X,Y and reduced velocity components U,V are related to their true values, indicated by x,y,u,v as follows:

$$X=x,\ Y=y\sqrt{1-M_\infty^2},\ U=\frac{u-u_\infty}{c^*-u_\infty},\ V=\frac{v}{(c^*-u_\infty)\sqrt{1-M_\infty^2}} \tag{2}$$

c^* being the critical sound speed. $U_p(X,Y)$ is the linearized Prandtl solution defined by

$$U_p(X,Y) = \frac{1}{\pi}\int_0^1 \frac{V_0(\xi)(X-\xi)}{(X-\xi)^2+Y^2}\,\mathrm{d}\xi \tag{3}$$

Niyogi[4,5] obtained an approximate solution of the singular integral equation (1), for shock-free flow as

$$U(X,Y) = (\sqrt{3}+1)\,[1-\{1-(\sqrt{3}-1)\,U_p(X,Y)\}^{1/2}] \tag{4}$$

Using the boundary condition at the profile

$$V(X) = V(X,0) = T\frac{\mathrm{d}f(X)}{\mathrm{d}X} \tag{5}$$

where T is the reduced thickness ratio, related to the thickness ratio τ as

$$T = \frac{\tau}{(1/M_\infty^* - 1)\sqrt{1-M_\infty^2}} \tag{6}$$

Equation (3) yields

$$U_p(X,Y) = \frac{A}{\pi}\int_0^1 \frac{\{\mathrm{d}h(X)/\mathrm{d}X\}(X-\xi)}{(X-\xi)^2+Y^2}\,\mathrm{d}\xi \tag{7}$$

where

$$A = \frac{1}{(1/M_\infty^* - 1)\sqrt{1-M_\infty^2}}\,;\quad h(x) = \tau f(x)$$

$h(x)$ is the profile shape.

The problem arises when $h(x)$ is not given analytically, and instead is prescribed by a set of numerical data. Our natural choice was then spline interpolation, which is capable of delivering results of adequate accuracy. Given a set of N mesh points $\{(x_i, y_i),\ i=1,\ldots,N\}$ describing the continuous profile shape, a cubic polynomial is chosen for the ith interval as

$$y = \alpha_i(x-x_i)^3 + \beta_i(x-x_i)^2 + \gamma_i(x-x_i) + \delta_i \tag{8}$$

Constants α_i, β_i, γ_i and δ_i are evaluated,[6] using the property that the cubics and their first and second derivatives are continuous (i.e., the condition to be required that both the slope, $\mathrm{d}y/\mathrm{d}x$, and the curvature, $\mathrm{d}^2y/\mathrm{d}x^2$, are the same for the pair of cubics that join at each point) at the pivotal points. The linearized solution $U_p(X,Y)$ is then found by simple integration as

$$U_p(X,Y) = \sum_{i=1}^{N} -\frac{1}{\pi A}\left[\frac{3\alpha_i}{2}\{(X_{i+1}-X)^2-(X_i-X)^2\}\right.$$

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